

Without attempting anything like a complete dynamical theory, which will require a large development of mathematics, I would point out the existence of a singular fundamental property of such granular media, which is not possessed by known fluids or solids. On perceiving something which resembles nothing within the limits of one's knowledge, a name is a matter of great difficulty. I have called this unique property of granular masses "dilatancy," because the property consists in a definite change of bulk, consequent on a definite change of shape or distortional strain, any disturbance whatever causing a change of volume and generally dilatation.

In the case of fluids, volume and shape are perfectly independent; and although in practice it is often difficult to alter the shape of an elastic body without altering its volume, yet the properties of dilatation and distortion are essentially distinct, and are so considered in the theory of elasticity. In fact there are very few solid bodies which are to any extent dilatable at all.

With granular media, the grains being sensibly hard, the case is, according to the results I have obtained, entirely different. So long as the grains are held in mutual equilibrium by stresses transmitted through the mass, every change of relative position of the grains is attended by a consequent change of volume; and if in any way the volume be fixed, then all change of shape is prevented.

In speaking of a granular medium, it is assumed to be in such a condition that the position of any internal particle becomes fixed, when the positions of the surrounding particles are fixed.

This condition is very generally fulfilled, but not always where there is friction; without friction it would be always fulfilled.

From this assumption it at once follows, that no grain in the interior can change its position in the mass by passing between the contiguous grains without disturbing these; hence, whatever alterations the medium may undergo, the same particle will always be in the same neighbourhood.

If, then, the medium is subject to an internal strain, the shapes of the internal groups of molecules will all be altered, the shape of each elementary group being determined by the shape of the surrounding particles. This will be rendered most intelligible by considering instances; that of equal spheres is the most general, and presents least difficulty.

A group of such spheres being arranged in such a manner that, if the external spheres are fixed, the internal ones cannot move, any distortion of the boundaries will cause an alteration of the mean density, depending on the distortion and the arrangement of the spheres. For example :—

If arranged as a pile of shot as in Fig. 2, which is an arrangement of tetrahedra and octahedra, the density of the media is $\frac{1}{\sqrt{2}} \frac{\pi}{3}$, taking the density of the sphere as unity.

If arranged in a cubical formation, as in Fig. 1, the density is $\frac{\pi}{6}$, or $\sqrt{2}$ times less than in the former case.

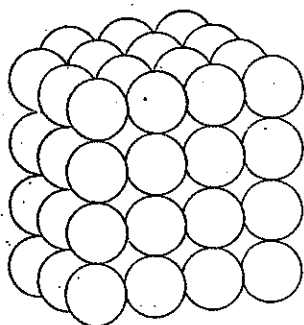


Fig. 1.

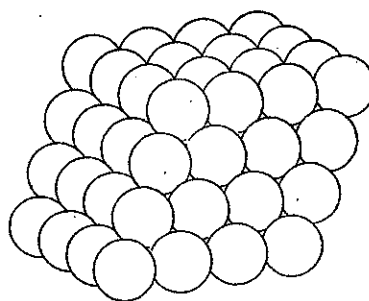


Fig. 2.

These arrangements are both controlled by the bounding spheres; and in either case the distortion necessitates a change of volume.

Either of these forms can be changed into the other by changing the shape of the bounding surface.

In both these cases the structure of the group is crystalline, but that is on account of the plane boundaries.

Practically, when the boundaries are not plane, or when the grains are of various sizes or shapes, such media consist of more or less crystalline groups having their axes in different directions, so that their mean condition is amorphous.

The dilatation consequent on any distortion for a crystalline group may be definitely expressed. When the mean condition is amorphous, it becomes difficult to ascertain definitely what the relations between distortion and dilatation are. But if, when at maximum density, the mean condition is not only amorphous but isotropic, a natural assumption seems to be, that any small contraction from the condition of maximum density in one direction, means an equal extension in two others at right angles.

As such a contraction in one direction continues, the condition of the medium ceases to be isotropic, and the relation changes until dilatation ceases. Then a minimum density is reached; after this, further contraction in the same direction causes a contraction of volume, which continues until

a maximum density is reached. Such a relation between the contraction in one direction, and the consequent dilatation, would be expressed by

$$e - 1 = e_1 \sqrt{\sin^2 \frac{a}{e_1}};$$

e being the coefficient of dilatation, a that of contraction, and e_1 the maximum dilatation; the positive root only to be taken.

The amorphous condition of minimum volume is a very stable condition; but there would be a direct relation between the strains and stresses in any other condition if the particles were frictionless and rigid.

If the particles were rigid, the medium would be absolutely without resilience, and hence the only energy of which it would be susceptible would be kinetic energy; so that, supposing the motion slow, the work done upon any group in distorting it would be zero. Thus, supposing a contraction in one direction and expansion at right angles, then if p_x be the stress in the direction of contraction, and p_y , p_z the stress at right angles, a being the contraction, b and c expansions,

$$p_x a + p_y b + p_z c = 0;$$

or, supposing $b = c$, $p_y = p_z$,

$$p_x a + p_y (a + c) = 0.$$

With friction the relation will be different; the friction always opposes strain, *i.e.* tends to give stability.

It is a very difficult question to say exactly what part friction plays; for although we may perhaps still assume without error,

$$\frac{p_y}{p_x} = \frac{1 - \sin \phi}{1 + \sin \phi},$$

where ϕ is the angle of repose, we cannot assume that $\tan \phi$ has any relation to the actual friction between the molecules.

The extreme value of ϕ is a matter of arrangement; as in the case of shot, which would pile equally well although without friction.

Supposing the grains rigid, the relations between distortion and dilatation are independent of friction; that is to say, the same distortion of any bounding surfaces must mean the same internal distortion whatever the friction may be.

The only possible effect of friction would be to render the grains stable under circumstances under which they would not otherwise be stable; and hence we might, with friction, be able to bring about an alteration of the boundaries other than the alteration possible without friction; and thus we

might possibly obtain a dilatation due to friction. How far this is the case can be best ascertained by experiment.

In the case of a granular medium, friction may always be relaxed by relieving the mass of stress, and any stability due to this cause would be shown by shaking the mass when in a condition of no stress.

But before applying this test, it is necessary to make perfectly sure that during the shaking the boundary spheres do not change position.

Another test of the effect of friction is, by comparing the relative dilatation and distortion with different degrees of friction. If the dilatation were in any sense a consequence of friction, it would be greater when the coefficient of friction between the spheres was greater. Where the granular mass is bounded by solid surfaces, the friction of the grains against these surfaces will considerably modify the results.

The problem presented by frictionless balls is much simpler than that presented in the case of friction. In the former case the theoretical problem may be attacked with some hope of success. With friction the property is most easily studied by experiment.

As a matter of fact, if we take means to measure the volume of a mass of solid grains more or less approximately spheres, the property of dilatancy is evident enough, and its effects are very striking, affording an explanation of many well-known phenomena.

If we have in a canvas bag any hard grains or balls, so long as the bag is not nearly full it will change its shape as it is moved about; but when the sack is approximately full, a small change of shape causes it to become perfectly hard. There is perhaps nothing surprising in this, even apart from familiarity; because an inextensible sack has a rigid shape when extended to the full, any deformation diminishing its capacity, so that contents which did not fill the sack at its greatest extension fill it when deformed. On careful consideration, however, many curious questions present themselves.

If, instead of a canvas bag, we have an extremely flexible bag of india-rubber, this envelope, when filled with heavy spheres (No. 6 shot), imposes no sensible restraint on their distortion; standing on the table it takes nearly the form of a heap of shot. This is, apparently accounted for by the fact that the capacity of the bag does not diminish as it is deformed. In this condition it really shows us less of the qualities of its granular contents than the canvas bag. But as it is impervious to fluid, it will enable me to measure exactly the volume of its contents.

Filling up the interstices between the shot with water, so that the bag is

quite full of water and shot, no bubble of air in it, and carefully closing the mouth, I now find that the bag has become absolutely rigid in whatever form it happened to be when closed.

It is clear that the envelope now imposes no distortional constraint on the shot within it, nor does the water. What, then, converts the heap of loose shot into an absolutely rigid body? Clearly the limit which is imposed on the volume by the pressure of the atmosphere.

So long as the arrangement of the shot is such that there is enough water to fill the interstices, the shot are free, but any arrangement which requires more room, is absolutely prevented by the pressure of the atmosphere.

If there is an excess of water in the bag when the shot are in their maximum density, the bag will change its shape quite freely for a limited extent, but then becomes instantly rigid, supporting 56 lb. without further change. By connecting the bag with a graduated vessel of water, so that the quantity which flows in and out can be measured, the bag again becomes susceptible of any amount of distortion.

Getting the bag into a spherical form, and its contents at maximum density, and then squeezing it between two planes, the moment the squeezing begins the water begins to flow in, and flows in at a diminishing rate until it ceases to draw more water.

The material in the bag is in a condition of minimum density under the circumstances. This does not mean that all the parts are in a condition of minimum density, because the distortion is not the same in all the parts; but some parts have passed through the condition of maximum, while others have not reached it, so that on further distortion the dilatations of the latter balance the contractions of the former. If we continue to squeeze, water begins to flow out until about half as much has run out as came in; then again it begins to flow in. We cannot by squeezing get it back into a condition of uniform maximum density, because the strain is not homogeneous. This is just what would occur if the shot were frictionless; so that it is not surprising to find that, using oil instead of water, or, better (on account of the india-rubber), a strong solution of soap and water, which greatly diminishes the friction, the results are not altered.

On measuring the quantities of water, we find that the greatest quantity drawn in is about 10 per cent. of the volume of the bag; this is about one-third of the difference between the volumes of the shot at minimum and maximum density.

$$\frac{1}{\sqrt{2}} : 1, \text{ or } 30 \text{ per cent. of the latter.}$$

On easing the bag it might be supposed that the shot would return to their initial condition. But that does not follow: the elasticity of form of the bag is so slight compared with its elasticity of volume, that restitution will only take place as long as it is accompanied with contraction of volume.

So long as the point of maximum volume has not been reached, approximate restitution follows quite as nearly as could be expected, considering that friction opposes restitution. But when the squeezing has been carried past the point of maximum volume, then restitution requires expansion; and this the elasticity of shape is not equal to accomplish, so that the bag retains its flattened condition. This experiment has been varied in a great variety of ways.

The very finest quartz sand, or glass balls $\frac{3}{4}$ inch in diameter, all give the same results. Sand is, on the whole, the most convenient material, and its extreme fineness reduces any effect of the squeezing of the india-rubber between the interstices of the balls at the boundaries; which effect is very apparent with the balloon bags, and shot as large as No. 6.

A well-marked phenomenon receives its explanation at once from the existence of dilatancy in sand. When the falling tide leaves the sand firm, as the foot falls on it the sand whitens, or appears momentarily to dry round the foot. When this happens the sand is full of water, the surface of which is kept up to that of the sand by capillary attraction; the pressure of the foot causing dilatation of the sand, more water is required, which has to be obtained either by depressing the level of the surface against the capillary attraction, or by drawing water through the interstices of the surrounding sand. This latter requires time to accomplish, so that for the moment the capillary forces are overcome; the surface of the water is lowered below that of the sand, leaving the latter white or dryer until a sufficient supply has been obtained from below, when the surface rises and wets the sand again. On raising the foot it is generally seen that the sand under the foot and around becomes momentarily wet; this is because, on the distorting forces being removed, the sand again contracts, and the excess of water finds momentary relief at the surface.

Leaving out of account the effect of friction between the balls and the envelope, the results obtained with actual balls, as regards the relation between distortion and dilatation, appear to be the same as would follow if the balls were smooth.

The friction at the boundaries is not important as long as the strain over the boundaries is homogeneous, and particularly if the balls indent themselves into the boundaries, as they do in the case of india-rubber. But with

a plane surface, the balls at the boundaries are in another condition from the balls within. The layer of balls at the surface can only vary its density from $2/\sqrt{3}$ to 1. This means that the layer of balls at a surface can slide between that surface and the adjacent layer, causing much less dilatation than would be caused by the sliding of an internal layer within the mass. Hence, where two parts of the mass are connected by such a surface, certain conditions of strain of the boundaries may be accommodated by a continuous stream of balls adjacent to the surface. This fact made itself evident in two very different experiments.

In order to examine the formation which the shot went through, an ordinary glass funnel was filled with shot and oil, and held vertical while more shot were forced up the spout of the funnel. It was expected that the shot in the funnel would rise as a body, expanding laterally so as to keep the funnel full. This seems to have been the effect at the commencement of the experiment; but after a small quantity had passed up it appeared, looking at the side of the funnel, that the shot were rising much too fast, for which, on looking into the top of the funnel, the reason became apparent. A sheet of shot adjacent to the funnel was rising steadily all round, leaving the interior shot at the same level with only a slight disturbance.

In another experiment one india-rubber ball was filled with sand and water; at the centre of this ball was another much smaller ball, communicating through the sides of the outer envelope by means of a glass pipe with an hydraulic pump. It was expected that, on expanding the interior ball by water, the sand in the outer ball would dilate, expanding the outer ball and drawing more water into the intervening sand. This it did, but not to the extent expected. It was then observed that the outer envelope, instead of expanding, generally bulged in the immediate neighbourhood of the point where the glass tube passed through it; showing that this tube acted as a conductor for the sand from the immediate neighbourhood of the interior ball to the outer envelope, just as the glass sides of the funnel had acted for the shot.

As regards any results which may be expected to follow from the recognition of this property of dilatancy,—

In a practical point of view, it will place the theory of earth-pressures on a true foundation. But inasmuch as the present theory is founded on the angle of repose, which is certainly not altered by the recognition of dilatancy, its effect will be mainly to show the real reason for the angle of repose.

The greatest results are likely to follow in philosophy, and it was with a view to these results that the investigation was undertaken.

The recognition of this property of dilatancy places a hitherto unrecognized mechanical contrivance at the command of those who would explain the fundamental arrangement of the universe, and one which, so far as I have been able to look into it, seems to promise great things, besides possessing the inherent advantage of extreme simplicity.

Hitherto no medium has ever been suggested which would cause a statical force of attraction between two bodies at a distance. Such attraction would be caused by granular media in virtue of this dilatancy and stress. More than this, when two bodies in a granular medium under stress are near together, the effect of dilatancy is to cause forces between the bodies, in very striking accordance with those necessary to explain coherence of matter.

Suppose an outer envelope of sufficiently large extent, at first not absolutely rigid, filled with granular media, at its maximum density. Suppose one of the grains of the media commences to grow into a larger sphere; as it grows, the surrounding medium will be pushed outwards radially from the centre of the expanding sphere. Considering spherical envelopes following the grains of the medium, these will expand as the grains move outwards. This fixes the distortion of the medium, which must be contraction along the radii, and expansion along all tangents.

The consequent amount of dilatation depends on the relation of distortion and dilatation, and on the arrangement of the grains in the medium. At first the entire medium will undergo dilatation, which will diminish as the distance from the centre increases. As the expansion goes on, the medium immediately adjacent to the sphere will first arrive at a condition of minimum density; and for further expansion this will be returning to a maximum density, while that a little further away will have reached a minimum. The effect of continued growth will therefore be, to institute concentric undulations of density from maximum to minimum density, which will move outwards; so that after considerable growth, the sphere will be surrounded with a series of envelopes of alternately maximum and minimum density, the medium at a great distance being at maximum density. At a definite distance from the centre of the sphere not more than

$$1.4R,$$

where R is the radius of the sphere, the density will be a minimum, and between this and the sphere there may be a number of alternations, depending on the relative diameters of the grains and the spheres.

The distance between these alternations will diminish rapidly as the sphere is approached. The distance of the next maximum is $1.2R$, the next minimum is given by $1.09R$, and the next maximum $1.06R$.

The general condition of the medium around a sphere which has expanded in the medium, is shown in Fig. 3, which has been arrived at on the supposition that the sphere is large compared with the grains.

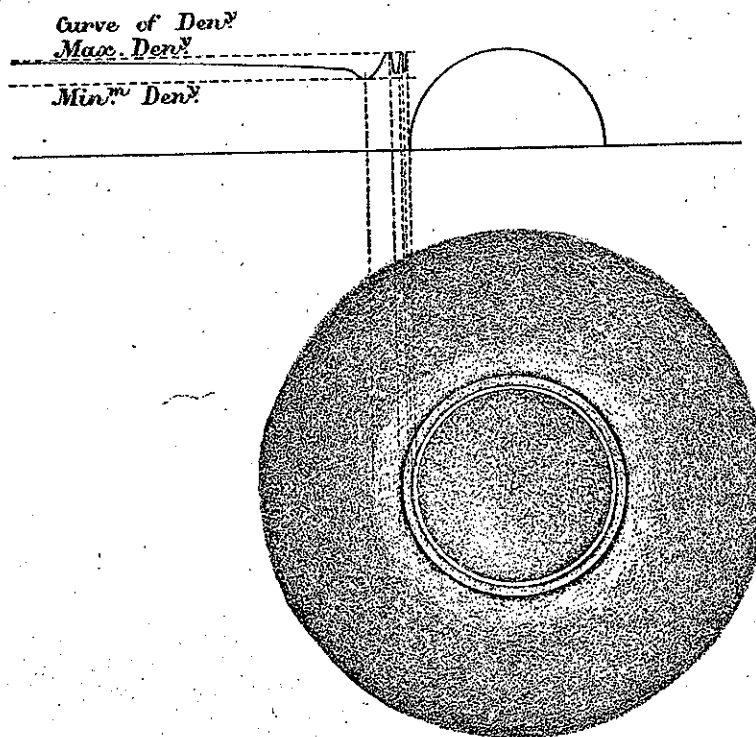


Fig. 3.

From a radius about $1.4R$ outwards the density gradually increases, reaching a maximum density at infinity; and at all distances greater than $1.8R$ the law is expressed by

$$\frac{de}{dr} = \frac{1}{r^n},$$

where n has some value greater than 3, depending on the structure of the medium.

Within the distance $1.4R$ the variation is periodic, with a rapidly diminishing period. In this condition, supposing the medium of unlimited extent and the sphere smooth, the sphere may move without causing further expansion, merely changing the position of the distortion in the medium; for the grains, slipping over the sphere, would come back to their original positions. It thus appears that smooth bodies would move without resistance, if the relation between the size of the grains and bodies is such, that the energy due to the relative motion of the grains in immediate proximity may

be neglected. The kinetic energy of the motion of the medium would be proportional to the volume of the ball, multiplied by the density of the medium, and the square of the velocity.

But the momentum might be infinite, supposing the medium infinite in extent, in which case a single sphere would be held rigidly fixed.

If we suppose two balls to expand instead of one, and suppose the distortion of the medium for one ball to be the same as if the other were not there, the result will be a compound distortion. Since, however, the dilatation does not bear a linear relation to the distortion, the dilatation resulting from the compound distortion will not be the sum of the dilatations for the separate distortions, unless we neglect the squares and products of the distortions as small.

Supposing the bodies so far apart that one or other of the separate distortions caused at any point is small, then, retaining squares and products, it appears that the resultant dilatation at any point will be less than the sum of the separate dilatations, by quantities which are proportional to the products of the separate distortions.

The integrals of these terms through the space bounded by spheres of radii R and L , are expressed by finite terms, and terms inversely proportional to L , which latter vanish if L is infinite. Thus, while the total separate dilatations are infinite, the compound dilatations differ from the sum of the separate by finite terms, and these are functions of the product of the volumes, and the reciprocal of the distance.

Assuming stress in the medium, the difference in the value of these finite terms for two relative positions of the bodies, multiplied by the stresses, represents an amount of work which must be done by the bodies on the medium in moving from one position to another.

To get rid of the difficulty of infinite extent of medium, if for the moment we assume the envelope sufficiently large and imposing a normal pressure upon the medium, then, since the work done will be proportional to the dilatation, the force between the bodies will be proportional to the rate at which this dilatation varies with the distance between them.

The force between the bodies would depend on the character of the elasticity, as well as on the dilatation.

It is not necessary to assume the outer envelope elastic; this may be absolutely rigid, and one or both the balls elastic.

In such case the two balls are connected by a definite kinematic relation. As they approach they must expand, doing work which is spent in producing

energy of motion; as they recede, the kinetic energy is spent in the work of compressing the balls.

As already stated, the momentum of the infinite medium for a single ball in finite motion may be infinite, and proportional to the product of the volume of the ball by the velocity; but with two balls moving in opposite directions, with velocities inversely as the masses, the momentum of the system is zero. Therefore such motion may be the only motion possible in a medium of infinite extent.

When the distance between the balls is of the same order as their dimensions, the law of attraction changes with the law of the compound dilatations, and becomes periodic, corresponding to the undulations of density surrounding the balls. Thus, before actual contact was reached, the balls would suffer alternate repulsion and attraction, with positions of equilibrium more or less stable between, as shown in Figs. 4 and 5.

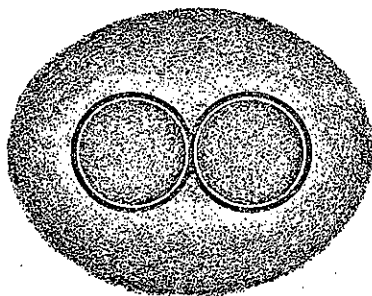


Fig. 4.

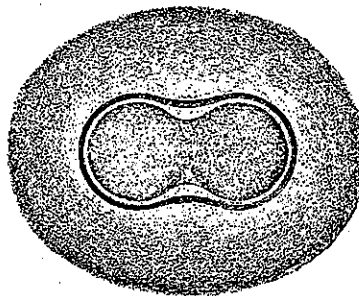


Fig. 5.

We have thus a possible explanation of the cohesion and chemical combination of molecules, which I think is far more in accordance with actual experience than anything hitherto suggested.

It was the observation of these envelopes of maximum and minimum density, which led me to look more fully into the property of dilatancy.

The assumed elasticity of the surrounding envelope, or of the balls, has only been introduced to make the argument clear.

The medium itself may be supposed to possess kinetic elasticity arising from internal distortional motion, such as would arise from the transmission of waves, in which the motion of the medium is in the plane of their fronts.

The fitness of a dilatant medium to transmit such waves is only less striking than its property of causing attraction, because in the first respect it is not unique.

But, as far as I can see, such transmission is not possible in a medium composed of uniform grains. If, however, we have comparatively large grains uniformly interspersed, then such transmission becomes possible. If, notwithstanding the large grains, the medium is at maximum density, the large grains will not be free to move without causing further dilatation; and it seems that the medium would transmit distortional vibrations, in which the distortions of the two sets of grains are opposite.

Such waves, although the motion would be essentially in the plane of the wave, would cause dilatation, just as waves in a chain cause contraction in the reach of the chain. They would in fact impart elasticity to the medium, exactly as, in the case of a slack chain having its ends fixed but otherwise not subject to forces, any lateral motion imparted to the chain will cause tension, proportional to the energy of disturbance divided by the slackness or free length of chain.

Distortional waves therefore, travelling through dilatant material which does not quite occupy the space in which it is confined when at maximum density, would render the medium uniformly elastic to distortion, but not in the same degree to compression or extension. The tension caused by such waves would depend on the gross energy of motion of the waves, divided by the total dilatation from maximum density consequent on the wave-motion. All such waves, whatever might be their length, would therefore move with the same velocity.

If, when rendered elastic by such waves, the medium were thrown into a state of distortion by some external cause, this would diminish the possible dilatation caused by the waves. Thus work would have to be done on the medium in producing the external distortion, which would be spent in increasing the energy of the waves. For instance, the separation of two bodies in such a medium, which, as already shown, would increase the statical distortion, would increase the energy of the waves and *vice versa*.

As far as the integrations have been carried for this condition of elasticity, it appears, with a certain arrangement of large and small grains, that the forces between the bodies would be proportional to the product of the volumes divided by the square of the distance; *i.e.* that the state of stress of the medium may be the same as Maxwell has shown must exist in the ether to account for gravity. We have thus an instance of a medium, transmitting waves similar to heat-waves, and causing force between bodies similar to the forces of gravitation and cohesion, in such a manner as to constitute a conservative system. More than this, by the separation of the two sets of grains, there would result phenomena similar to those resulting from the separation of the two electricities. The observed conducting power of a continuous surface for the grains of a medium, closely resembles the

conduction of electricity. And such a composite medium would be susceptible of a state in which the arrangement of the two sets of grains were thrown into opposite distortions, which state, so far as it has yet been examined, appears to coincide with the state of a medium necessary to explain electrodynamic and magnetic phenomena according to Maxwell's theory.

In this short sketch of the results which it appears to me may follow from the recognition of the property of dilatancy, I have not attempted to follow the exact reasoning even so far as I have carried it.

In the preliminary acceptance of a theory, the mind must be guided rather by a general view of its adaptability, than by its definite accordance with some out of many observed facts. And as it seems, after a preliminary investigation, that in space filled with discrete particles, endowed with rigidity, smoothness, and inertia, the property of dilatancy would cause amongst other bodies, not only one property, but all the fundamental properties of matter, I have, in pointing out the existence of dilatancy, ventured to call attention to this dilatant or kinematic theory of ether, without waiting for the completion of the definite integrations, which must take long, although it is by these that the fitness of the hypotheses must be eventually tested.

51.

EXPERIMENTS SHOWING DILATANCY, A PROPERTY OF
GRANULAR MATERIAL, POSSIBLY CONNECTED WITH
GRAVITATION.

[From the "Proceedings of the Royal Institution of Great Britain."]

(Read February 12, 1886.)

IN commencing this discourse, the author said, My principal object to-night is to show you certain experiments which I have ventured to think would interest you on account of their novelty, and of their paradoxical character. It is not, however, solely or chiefly on account of their being curious that I venture to call your attention to them. Let them have been never so striking, you would not have been troubled with them, had it not been that they afford evidence of a fact of real importance in mechanical philosophy.

This newly recognised property of granular masses, which I have called dilatancy, will, it may be hoped, be rendered intelligible by the experiments, but it was not by these experiments that it was discovered.

This discovery, if I may so call it, was the result of an attempt to conceive the mechanical properties a medium must possess, in order that it might fulfil the functions of an all-pervading ether—not only in transmitting waves of light, and refusing to transmit waves like those of sound, but in causing the force of gravitation between distant bodies, and actions of cohesion, elasticity, and friction between adjacent molecules, together with the electric and magnetic properties of matter, and at the same time allowing the free motion of bodies.

It will be well known to those who attend the lectures in this room, that although a vast increase has been achieved in knowledge of the actions called

the physical properties of matter, we have as yet no satisfactory explanation as to the *prima causa* of these actions themselves; that to explain the transmission of light and heat, it has been found necessary to assume space filled with material possessing the properties of an elastic jelly, the existence of which, though it accounts for the transmission of light, has hitherto seemed inconsistent with the free motion of matter, and failed to afford the slightest reason for the gravitation, cohesion, and other physical properties of matter. To explain these, other forms of ether have been invented, as in the corpuscular theory and the celebrated hypothesis of La Sage, the impossibilities of which hypotheses have been finally proved by the late Professor Maxwell, to whom we owe so much of our definite knowledge of the fundamental physics. Maxwell insisted on the fact, that even if each of the physical properties could be explained by a special ether, it would not advance philosophy, as each of these ethers would require another ether to explain its existence, *ad infinitum*. Maxwell clearly contemplated the existence of one medium, but it was a medium which would cause not one but all the physical properties of matter. His writings are full of definite investigations as to what the mechanical properties of this ether must be, to account for the laws of gravitation, electricity, magnetism, and the transmission of light, and he has proved very clear and definite properties, although, as he distinctly states, he was unable to conceive a mechanism which should possess these properties.

As the result of a long-continued effort to conceive a mechanical system possessing the properties assigned by Maxwell, and further, which would account for the cohesion of the molecules of matter, it became apparent that the simplest conceivable medium—a mass of rigid granules in contact with each other—would answer not one but all the known requirements, provided the shape and mutual fit of the grains were such, that while the grains rigidly preserved their shape, the medium should possess the apparently paradoxical, or anti-sponge property, of swelling in bulk as its shape was altered.

I may here remark, that if ether is atomic or granular, that it should be a mass of grains holding each other in position by contact, like the grains in the sack of corn, is one of only two possible conceptions; the other being that of La Sage, or the corpuscular theory that the grains are free like bullets, moving in space in all directions.

Nor, in spite of its paradoxical sound, is there any great difficulty of conceiving the swelling in bulk. When the grains are in contact, it appears at once that the mechanical properties of the medium must be to some extent affected by the shape and fit of the grains. And having arrived at the conclusion, that in order to act the part of ether, this shape and fit must be such

that the mass could not change its shape, without changing its volume or space occupied, the next thing was to see what possible shape could be given to the grains, so that while these rigidly preserved their shape, the medium might possess this property of dilatancy.

It was obvious that the grains must so interlock, that when any change of shape of the mass occurred, the interstices between the grains should increase. This would be possessed by grains shaped to fit into each other's interstices in one particular arrangement.

In an ordinary mass of brickwork or masonry well bonded without mortar, the blocks fit so as to have no interstices; but if the pile be in any way distorted, interstices appear, which shows that the space occupied by the entire mass has increased. (*Shown by a model.*)

At first it appeared that there must be something special and systematic, as in the brick wall, in the fit of the grain of ether, but subsequent consideration revealed the striking fact, *that a medium composed of grains, of any possible shape, possessed this property of dilatancy, so long as one important condition was satisfied.*

This condition is, that the medium should be continuous, infinite in extent, or that the grains at the boundary should be so held as to prevent a rearrangement commencing. All that is wanted is a mass of hard smooth grains, each grain being held by the adjacent grains, and the grains on the outside prevented from rearranging.

Smooth hard spheres arranged as an ordinary pile of shot are in their closest order, the interstices occupying a space about one-third that occupied by the spheres themselves. By forcing the outside shot so as to give the pile a different shape, the inside spheres are forced by those on the outside, and the interstices increase. Thus by shaping the outside of the pile, the interstices may be increased to any extent, until they occupy about nine-tenths of the volume of the spheres: this is the most open formation. A further change of shape in the same direction causes a contraction of the interstices, until a minimum volume is reached, and then again an expansion, and so on. The point to be realised is, that in any of these arrangements, if the whole of the spheres on the outside of the group are fixed, those inside will be fixed also. (*Shown by a model.*)

An interior portion of a mass of smooth hard spheres therefore cannot have its shape changed by the surrounding spheres, without altering the room it occupies, and the same is true for any granular mass, whatever be the shape of the grains.

Considering the generality of this conclusion, the non-discovery of this property as existing in tangible matter, requires a word of explanation.

The physical properties of elasticity, adhesion, and friction, so far render the molecules of ordinary matter incapable of behaving as a system of parts with the sole property of keeping their shape, and so prevent evidence of dilatancy in solids and fluids. This is quite consistent with dilatancy in the ether, for the properties of elasticity, cohesion, and friction, in tangible matter, are due to the presence of the ether, so that it would be illogical for the elementary atoms of the ether to possess these properties.

This, although a sufficient reason why dilatancy has not been recognised as a property of solid and fluid matter, does not explain its non-existence in masses of solid, hard, free grains, as of corn, shot, and sand. To understand why it has not been observed in these, it must be remembered that, to ordinary observation, these present only an outside appearance, and that the condition essential for dilatancy, that the outside grains should not be free to rearrange, is seldom fulfilled. Also these granular forms of matter, though commonplace, have not been the subjects of physical research, and hence such evidence as they do afford has escaped detection.

Once, however, having recognised dilatancy as a universal property of granular masses, it was obvious that if evidence of it was to be sought from tangible matter, it must be sought in what have hitherto been the most commonplace and least interesting arrangements. That an important geometrical and mechanical property of a material system should have been hidden for thousands of years, even in sand and corn, is such a striking thought, that it required no little faith in mechanical principles to undertake the search for it, and although finding nothing but what was strictly in accordance with the conclusions previously arrived at, the evidence obtained of this long-hidden property was as much a matter of visual surprise to the lecturer, as it can be to any of the audience.

To render the dilatancy of a granular mass evident, it was necessary to accomplish two things: (1) the outside grains must be controlled so that they could not rearrange, and this without preventing change of shape and bulk of the mass; (2) the changes of bulk or volume of the mass, or of the interstices between the grains, must be rendered evident by some method of measurement which did not depend on the shape of the mass.

A very simple means—a *thin india-rubber* envelope or boundary—answered both these purposes to perfection. The thin india-rubber closed over the outside grains sufficiently to prevent their change of position, and the impervious character of the bag allowed of a continuous measure of the volume of the contents, by measuring the quantity of air or water necessary to fill the interstices.

Taking an india-rubber bag which will hold six pints of water, without stretching, and having only a small tubular aperture, getting it quite dry,

and putting into it six pints of dry sea sand, such as will run in an hour-glass, sharp river sand, dry corn, shot or glass marbles, it presents no very striking appearance, but all the same when filled with any of these materials, it cannot have its form changed, as by squeezing between two boards, without changing its volume. These changes of volume are not sufficient to be noticeable while the squeezing is going on, but they may be rendered apparent. It is sufficient to do this with the bag full of clean dry Calais sand, such as is used in an hour-glass.

The tube from the bag is connected with a mercurial pressure-gauge, so that the bag is closed by the mercury.

The actual volume occupied by the quartz grains is four and a half pints. The remaining space, one and a half pints, is occupied by the interstices between the grains in their closest order; these interstices are full of air, so that three-quarters of the bag are occupied by quartz, and one-quarter by air. Since the bag is closed, and no more air can get in, if interstices are increased from one pint and a half to two pints, the air must expand, and its pressure will fall from that of the atmosphere to three-quarters of an atmosphere. As soon as squeezing begins, the mercury rises on the side connected with the bag, and steadily rises as the bag flattens, until it has risen seven inches, showing that the bag has increased in capacity by half a pint, or one-twelfth of its initial capacity.

That by squeezing a porous mass like sand we should diminish the pressure of the air in the pores is paradoxical, and shows the anti-sponginess of the granular material; had there been a sponge in the bag, the pressure of the air would have increased with the squeezing.

This experiment has been mainly introduced to prevent a possible impression that the fluid filling the interstices has anything to do with the dilatation besides measuring it.

Water affords a more definite measure of volume than air.

Taking a small india-rubber bottle with a glass neck full of shot and water, so that the water stands well into the neck. If instead of shot the bag were full of water, or had anything of the nature of a sponge in it, when the bag was squeezed the water would be forced up the neck. With the shot the opposite result is obtained; as I squeeze the bag, the water decidedly shrinks in the neck.

This experiment, which you see is on a very small scale, was not designed to show to an audience; it was the original experiment which was made for my own satisfaction, when the idea of dilatancy first presented itself. The result, but for the knowledge of dilatancy, would appear paradoxical, not to say magical. When we squeeze a sponge between two planes, water

is squeezed out; when we squeeze sand, shot, or granular material, water is drawn in.

Taking a larger apparatus, a bag which holds six pints of sand, the interstices of which are full of water without any air—the glass neck being graduated so as to measure the water drawn in. On squeezing the bag with a large pair of pincers, a pint of water is drawn from the neck into the bag. This is the maximum dilatation; the grains of sand are now in the most open order into which they can be brought by this squeezing; further squeezing causes them to take closer order, the interstices diminish, and the water runs out into the vessel, and for still further squeezing is drawn back again, showing that as the change of form continues, the medium passes through maximum and minimum dilatations.

This experiment may be repeated with granules of any size or shape, provided they are hard, and shows the universality of dilatancy.

Although not more definite, perhaps more striking evidence of dilatancy is afforded by the means which the non-expansibility of water affords of limiting the volume of the bag. An impervious bag full of sand and water without air cannot have its contents enlarged without creating a vacuum inside it—the interstices of the sand are therefore strictly limited to the volume of the water inside it, unless forces are brought to bear sufficient to overcome the pressure of the atmosphere and create a vacuum. Since then, owing to this property of dilatancy, the shape of a granular mass at its greatest density cannot change without enlarging the interstices, if we prevent this enlargement by closing the bag we prevent change of shape.

Taking the same bag, the sand being at its closest order—and closing the neck so that it cannot draw more water. A severe pinch is put on the bag, but it does not change its shape at all; the shape cannot alter without enlarging the interstices, which cannot enlarge without drawing more water, and this is prevented. To show that there is an effort to enlarge going on, it is only necessary to open a communication with a pressure-gauge, as in the experiment with air. The mercury rises on the side of the bag, showing when the pinch is hardest (about 200 lbs. on the planes) that the pressure in the bag is less by 27 inches of mercury than the pressure of the atmosphere; a little more squeezing and there is a vacuum in the bag. Without a knowledge of the property of dilatancy such a method of producing a vacuum would sound somewhat paradoxical. Opening the neck to allow the entrance of water, the bag at once yields to a slight pressure, changing shape, but this change at once stops when the supply is cut off, preventing further dilatation.

In these experiments neither the thickness of the bag, nor the character of the fluid, has anything to do with the dilatation of the contents.

considered as forming an interior group of a continuous medium, the bag merely controlling the outside members as they would be controlled by surrounding grains, and the fluid merely measuring or limiting the volume of the interstices.

It has, however, been absence of such control of the outside grains, and such means of measuring the volume of the interstices, that has prevented the dilatancy revealing itself as a general mechanical property of granular material; as a *mechanical* property, because dilatancy has long been known to those who buy and sell corn. It is seldom left for the philosopher to discover anything which has a direct influence on pecuniary interests; and when corn was bought and sold by *measure*, it was in the interest of the vendor to make the interstices as large as possible, and of the vendee to make them as small; of the vendor to make the corn lie as lightly as possible, and of the vendee to get it as dense as possible. These interests are obvious; but the methods of getting corn dense and light are *paradoxical* when compared with the methods for other material. If we want to get any elastic material light we shake it up, as a pillow or a feather bed, or a basket of dried fruit; to get these dense we squeeze them into the measure. With corn it is the reverse; it is no good squeezing it to get it dense; if we try to press it into the measure we make it light—to get it dense we must shake it—which, owing to the surface of the measure being free, causes a rearrangement in which the grains take the closest order.

At the present day the measure for corn has been replaced by the scales, but years ago corn was bought and sold by measure only, and measuring was then an art which is still preserved. It is understood that the corn is to be measured light, and the method employed is now seen to have made use of the property of dilatancy. The measure is filled over full and the top struck with a round pin called the strake or strickle. The universal art is to put the strake end on into the measure before commencing to fill it. Then when heaped full, to pull the strake gently out and strike the top; if now the measure be shaken it will be seen that it is only nine-tenths full.

Sand presents many striking phenomena well known but not hitherto explained, which are now seen to be simply evidence of dilatancy.

Every one who walks on the strand must have been painfully struck with the difference in the firmness and softness of the sand at different times; letting alone when it is quite dry and loose. At one time it will be so firm and hard that you may walk with high heels without leaving a footprint; while at others, although the sand is not dry, one sinks in so as to make walking painful. Had you noticed you would have found that the sand is firm as the tide falls, and becomes soft again after it has been left dry for some hours. The reason for this difference is exactly the same as

that of the closed bags with water and air in the interstices of the sand. The tide leaves the sand, though apparently dry on the surface, with all its interstices perfectly full of water, which is kept up to the surface of the sand by capillary attraction; at the same time the water is percolating through the sand from the sands above, where the capillary action is not sufficient to hold the water. When the foot falls on this water-saturated sand, it tends to change its shape, but it cannot do this without enlarging the interstices—without drawing in more water. This is a work of time, so that the foot is gone again before the sand has yielded. If you stand still, you will find that your feet sink more or less, and that when you move, the sand becomes wet all round the space you stood on, which is the excess of water you have drawn in, set free by the sand regaining its densest form.

One phenomenon attending walking on firm sand is very striking; as the foot falls, the sand all round appears to shoot white or dry momentarily, soon becoming dark again. This is the suction into the enlarging interstices below the foot, which for the moment depresses the capillary surface of the water below that of the sand.

After the tide has left the sand for a sufficient time, the greater part of the water has run out of the interstices, leaving them full of air, which by expanding allows the interstices to enlarge, and the foot to sink in far enough to make walking unpleasant.

If we walk on sand under water, it is always more or less soft, for the interstices can enlarge, drawing in water from above.

The firmness of the sand is thus seen to be due to the interstices being full of water, and to the capillary action or surface tension of the water at the surface of the sand. This capillary action will hold the water up in the sand for some inches or feet, according to the fineness of the sand. This is shown by a somewhat striking experiment. If sand running in a stream from a small hole in the bottom of a vessel, as in an hour-glass, fall into a vessel containing a slight depth of water, the sand at first forms an island, which rises above the water. The sand which then falls on the top of this island is dry as it falls, but capillary action draws up the water which fills the interstices and gives the sand coherence. The island grows vertically, very fast, and assumes the form of a column, sometimes with branches like a tree or a fern, some inches or even a foot high. The strength of these consists in the surface tension of the water preventing air from being drawn in to enlarge the interstices, which therefore cannot change shape; it is therefore another evidence of dilatancy.

By substituting an impervious envelope for the surface of water, firmness of sand saturated with water may be rendered very striking.

Thin india-rubber balloons, which may be easily expanded with the mouth, afford an almost transparent envelope.

Taking one containing about six pints of sand and water, closed without air, there being more water than will fill the interstices at the densest, but not enough to allow them to enlarge to the full extent. When standing on the table, the elasticity of the envelope gives it a rounded shape. The sand has settled down to the bottom, and the excess of water appears above the sand, the surface of which is free. The bag may be squeezed and its shape altered, apparently as though it had no firmness, but this is only so long as the surface is free. But taking it between two vertical plates and squeezing, at first it submits, apparently without resistance, when all at once it comes to a dead stop. Turning it on to its side, a 56-lb. weight produces no further alteration of shape; but on removing the weight, the bag at once returns to its almost rounded shape.

Putting the bag now between two vertical plates, and slightly shaking while squeezing, so as to keep the sand at its densest, while it still has a free surface, it can be pressed out until it is a broad flat plate. It is still soft as long as it is squeezed, but the moment the pressure is removed, the elasticity of the bag tends to draw it back to its rounded form, changing its shape, enlarging the interstices, and absorbing the excess of water; this is soon gone, and the bag remains a flat cake with peculiar properties. To pressures on its sides it at once yields, such pressures having nothing to overcome but the elasticity of the bag, for change of shape in that direction causes the sand to contract. To radial pressures on its rim, however, it is perfectly rigid, as such pressures tend further to dilate the sand; when placed on its edge, it bears one cwt. without flinching.

If, however, while supporting the weight it is pressed sufficiently on the sides, all strength vanishes, and it is again a rounded bag of loose sand and water.

By shaking the bag into a mould, it can be made to take any shape; then, by drawing off the excess of water and closing the bag, the sand becomes perfectly rigid, and will not change its shape without the envelope be torn; no amount of shaking will effect a change. In this way bricks can be made of sand or fine shot full of water and the thinnest india-rubber envelope, which will stand as much pressure as ordinary bricks without change of shape; also permanent casts of figures may be taken.

I have now shown, as fully as time will allow, the experiments which afford evidence of the existence of the property of dilatancy, and how it explains natural phenomena hitherto but little noticed.

Beyond affording evidence of the existence of the property dilatancy,

these experiments have no direct connection with gravitation or the physical properties of matter.

These properties cannot be deduced by direct experiment on granular material, for the simple reason that the grains of the medium which constitutes the ether must be free from friction, while the grains with which we work are subject to friction. These properties can only be deduced by mathematical reasoning, into which I will not drag you to-night. I will merely point out two or three facts, which may serve to convey an idea of how dilatancy should have such a bearing on the foundation of the universe.

If you look at this diagram, you see it represents a ball surrounded by a continuous mass of grain, the density of the grains being indicated by the depth of colour. If that ball were to grow in volume, it would have to push out the medium on all sides, and in that way it would distort the groups of grains, or change their form, causing the interstices to increase; those nearer the ball would be distorted more than those further away. Then the interstices of these would grow the most rapidly, and those adjacent to the ball would first come to their openest order for further growth; these would contract somewhat, those a little further away would reach the openest order, and if the process of growth steadily continued, we should have a series of undulations of density, commencing at the ball and moving outwards; the first of these waves of open order would not, however, get beyond half the diameter of the ball away. The diagram represents the interstices that would result, if a single grain of the material had grown to the size of the ball, pushing the medium out before it. It is not necessary that the ball should have grown, to produce this result; however the ball were originally placed, if it were moved away from its original place, it would assume this arrangement, and with this arrangement it would be free to move. Now, although I cannot attempt to enter upon the relation between the density of the medium, and the force of attraction between two bodies in it, I may call your attention to this fact, that the dilatation as calculated, varies exactly as the force of gravitation, inversely as the square of the distance from an infinite distance till close to the ball, and then goes through several undulations, corresponding exactly to the variations in the attraction of bodies necessary to explain the elasticity and cohesion of molecules. As is shown in the other diagrams, these undulations in density, which may be experimentally produced, not only appear to afford a clear explanation of cohesion, but are the only suggestion of an explanation ever made. And further, similar undulations have been found necessary to explain one of the phenomena of light. My reason for calling your attention to them was partly an experiment, which, although not the most striking, is the most advanced experiment in the direction of dilatancy.

The apparatus is that represented in the diagram; the medium is contained in the large elastic bag; in the middle of this bag is a small hollow elastic ball, which can be expanded by water forced in through a tube passing through the medium and outside ball; the quantity of water which passes in is measured by a mercury gauge, the water being forced in by the pressure of the mercury. The medium between the two balls is sand and water, and is connected with a gauge, the water drawn from which measures the dilatation.

The full pressure of 30 inches is on the interior ball, but produces no expansion, because the medium outside cannot dilate, as the supply of water is now cut off; opening the tap to admit water to the outer ball, it at once draws water. It has now drawn 3 oz.; in the meantime the mercury has fallen, showing that an ounce and a half was admitted to the interior ball, the expansion of which drew the water into the outer envelope. This experiment is not striking, but it is definite, and enables us to measure the dilatation consequent on a given distortion.

It is impossible for me to go further into this explanation, so I will merely state that the ability of the grains of a medium to slide over a smooth surface has been experimentally shown to produce phenomena closely resembling the conduction of electricity, to complete which it is only necessary to construct the medium of two different sorts of grains, different in size or different in shape, the separation of which would afford the two electricities, and be a simple way out of the difficulty hitherto found in explaining the non-exhaustibility of the electricity in a body. Hitherto the two electric fluids have been supposed to reside together in the matter of the machine, which, however much has been withdrawn, has never shown signs of exhaustion. In the dilatant hypothesis, these electricities are the two constituents of the ether which the machine separates, and it is worth noticing that the ordinary electrical machine resembles in all essential particulars the machines used by seedsmen for separating two kinds of seed, trefoil and rye-grass, which grow together: as long as there is a supply of the mixture, the machine is never exhausted.

This dilatant hypothesis of ether is very promising, although it cannot be put forward as proved until it has been worked out in detail, which will take long. In the meantime it is put forward mainly to excite interest in the property of dilatancy, to the discovery of which it has led. This property, now that it has once been recognised, is quite independent of any hypothesis, and offers a new field for philosophical and mathematical research quite independent of the ether.